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Gravitational Repulsion and Dirac Antimatter

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Based on an analogy with electron and hole dynamics in semiconductors, Dirac's relativistic electron equation is generalized to include a gravitational interaction using an electromagnetic-type approximation of the gravitational potential. With gravitational and inertial masses decoupled, the equation serves to extend Dirac's deduction of antimatter parameters to include the possibility of gravitational repulsion between matter and antimatter. Consequences for general relativity and related "antigravity" issues are considered, including the nature and gravitational behavior of virtual photons, virtual pairs, and negative-energy particles. Basic cosmological implications of antigravity are explored—in particular, potential contributions to inflation, expansion, and the general absence of detectable antimatter. Experimental and observational tests are noted, and new ones suggested.

1. INTRODUCTION

Soon after Dirac (1928a,b) derived his relativistic electron equation, he deduced the existence of positrons from the equation's negative-energy (or negative-mass) solutions,[2] $E = -(m^2c^4 + p^2c^2)^{1/2} = -mc^2(1 - v^2/c^2)^{-1/2}$, where m is the electron rest mass and $\mathbf{p}$ is its momentum. To circumvent the threat of wholesale annihilation of electrons via radiative transitions into negative-energy states, he assumed that these states were (nearly) all occupied, constituting a "Fermi sea" of negative-energy electrons. Dirac (1931, 1958) inferred that absorption of a sufficiently energetic photon could raise an electron out of the sea, leaving a "hole," an unoccupied state, interpreted as a positive-energy, positively charged electron or "positron."[3]

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[2] See Sections 3 and 6 for an interpretation of the concept of negative rest mass.

[3] See especially Dirac (1958), Section 73, pp. 273–275, "Theory of the Positron." The author's extension of Dirac's positron model to include gravity is based on Dirac (1958), Chapter XI, section on the "Relativistic Theory of the Electron."

Soon afterward, positrons were observed in cosmic rays by Anderson (1932); but, like other components of "antimatter" (the mirror image of normal atomic matter), they are rare in our part of the universe, produced only in high-energy interactions. For example, no known naturally occurring isotopes decay by positron emission; the first such decay, observed by Curie and Joliot (1934) [cited in McGervey (1971), p. 490], was also the first artificially induced radioactivity, via α-bombardment of ${}_5B^{10}$.

Dirac described a positive-energy electron surrounded by a uniform, infinitely dense sea of negative-energy electrons, each of which exerts a repulsive electrostatic (ES) force on the positive-energy electron; but the uniformity of the sea guarantees that $\mathbf{div}\ \mathbf{E} = 0$,[4] so that, on the average, there will be no net ES force on the electron. As in solid-state theory, a vacancy or "hole" in the vacuum (sea) will effectively attract the positive-energy, negatively charged electron.[5]

Dirac believed that the "negative" of this picture—with the sea composed of positrons and the holes representing electrons—should be equally valid. This symmetry suggests that holes should have the same mobility within the sea as free particles and antiparticles exhibit above it. The model would be expected to hold in some degree for other elementary fermions; the form used in quantum chromodynamics (QCD) to describe baryons will not be considered here.

The negative-energy sea makes Dirac's single-electron theory in some sense a many-body theory; thus, it is usually translated into second-quantization representation. In the process, however, not only are infinite self-energies lost, but so is some of the heuristic value of the original formulation. It is instructive to return to Dirac's antimatter derivation, extending it to include gravitation. Though this gravitational interaction is, under normal circumstances, exceedingly small compared to ES interactions for charged fermions, it may play a crucial role in cosmology, quantum gravity, and particle physics.

2. TWO SCENARIOS

Before tackling Dirac's equation, let us explore two scenarios that predict diametrically opposed gravitational characteristics for positrons, and give the

[4] Dirac's interpretation of the Laplace equation is that ES force is produced by charge excess or deficit relative to the vacuum charge. See Section 2 below for an alternative approach to this issue, which considers how the vacuum charge might be compensated by a positively charged matrix in which it may be embedded.

[5] For discussions of hole theory from the viewpoint of semiconductors and other aspects of solid-state physics see, e.g., Harrison (1979) (esp. Chapter II, Section 6, and problem 4.3, p. 456; for a discussion of Cooper pairs see pp. 495ff), Kittel (1971), Madelung (1978), and Mattuck (1976) (esp. Section 7.5 for a simple, lucid discussion of particle–hole formalism), and references therein.

reasons for selecting the model used in this paper: The conventional scenario, consistent with the equivalence principle (EP), assumes that electrons and positrons (indeed, all particles and photons) are mutually attractive gravitationally; i.e., in the absence of all other forces, all particles (and photons) would accelerate toward one another. As will be shown, this implies that negative-energy electrons should have negative gravitational and inertial mass, which is equivalent to the usual assumption that holes (positrons) have positive gravitational and inertial mass.

It will be argued that an alternative scenario, consistent with the solid-state analogy proposed below (viz., that negative- and positive-energy electrons have gravitational masses with the same sign, while holes have negative gravitational mass) should not be ruled out *a priori*. Generally, the gravitational interactions of negative-energy electrons are not considered, because they enter our calculations (except for vacuum polarization) only in their absence, viz., as holes or positrons. It will be shown, using a naive Newtonian model of gravitation and inertia, that negative-energy electrons (which, according to the usual relativistic formulation, also have negative inertial mass) should behave under either scenario in a manner inconsistent with the intuitive picture of universal mutual attraction, but consistent with the usual picture of electrons in semiconductors.

For normal matter (e.g., an electron) of mass m_1 to exhibit gravitational attraction, the Newtonian gravitational interaction (with normal matter of mass m_2) must be negative, i.e., $F = -Gm_1m_2/r^2 < 0$. The gravitational mass of the electron is conventionally positive; if it is attracted by every other mass in the universe, then every gravitational mass must be positive, since $\mathbf{F} = m_1\mathbf{a}$. Negative-energy electrons, however, have negative inertial mass, since $m = E/c^2 < 0$; in that case, because $\mathbf{F} = m\mathbf{a}$, the component of their acceleration due to gravity must be directed away from normal matter (including a positive-energy electron), which contradicts the assumption that they are mutually attracted gravitationally in the usual sense.

On the other hand, suppose that the normal electron is gravitationally repelled by the negative-energy electron. Then the gravitational mass of the latter must be negative, since $-Gm_1m_2/r^2$ must be positive for the normal electron to be repelled; but in that case, the negative-energy electron (whose inertial mass is also negative) accelerates *toward* the normal electron. Therefore, if acceleration is the operational criterion, positive- and negative-energy electrons are neither mutually repelled nor mutually attracted gravitationally. Notice also that between two negative-energy electrons, the Newtonian gravitational interaction must appear attractive ($F < 0$), while the law of inertia indicates mutual repulsion ($m < 0$).

This sort of counterintuitive behavior is relatively familiar in solid-state physics, where the effective inertial mass of an electron or hole is determined by the total (mainly EM) interaction between the electrons and the lattice. (Electrostatic interactions among electrons are typically ignored in this approximation, although their energies and momenta obviously depend on correlations anchored in the Pauli exclusion principle, and thus on electron spin.) The "bare" gravitational mass (m_g) of an electron interacts with an external gravitational field ($\mathbf{g}$) to produce a force ($m_g\mathbf{g}$) on that electron, whose response to the force (i.e., its acceleration) is conditioned by its effective inertial mass (m^*), a parameter quite distinct from the gravitational mass in the law of inertia: $\mathbf{F} = m_g\mathbf{g} = m^*\mathbf{a}$. In any case, it is clearly problematic to assume that *all* particles and photons would accelerate toward one another gravitationally in the absence of other interactions.

Although we have no idea what the underlying matrix of space might be, we may suppose that the negative-energy electrons populating the nearly degenerate sea posited by Dirac exhibit negative inertial mass for a reason analogous to that which causes the negative effective mass of, for example, electrons near the top of the valence band in semiconductors; viz., these electrons are in a potential well produced by a positively charged "lattice." Indeed, it is difficult to imagine a simpler heuristic model to illustrate what negative energy might mean.

The coexistence of such a positive substrate with Dirac's sea would certainly be more consistent with Gauss' law than the existence of an uncompensated sea of negative charge; the latter gives a nonvanishing div $\mathbf{E}$ for the vacuum, even though no force (except isotropic pressure) is possible for an infinite, uniform charge distribution. Polarization of the vacuum, as exhibited in Lamb-shift experiments (Lamb and Retherford, 1957), would still occur in the presence of an observable electron; in fact, it would be more self-consistent intuitively than the Dirac picture alone, since it is not at all clear how a free electron could polarize an otherwise uniform distribution of (moving) negative-energy electrons of infinite extent.

If this analogy is valid, then it provides a basis, consistent with Dirac theory, for supposing that negative-energy electrons are normal electrons that happen to be bound within a matrix characterized by a "work function" of $2mc^2$, while positrons are just vacancies (holes) in negative-energy states. As mentioned above, an external gravitational field interacts with the "bare" gravitational mass (m_g) of the electron in a conduction or valence band of a solid, not with its "clothed" effective inertial mass (m^*), which may be positive or negative, depending on its energy and momentum (or wave number, for a Bloch electron). The inertial mass is associated with the response side of the equation ($m^*\mathbf{a}$), the gravitational mass with the force side ($m_g\mathbf{g}$).

Thus, it would be consistent with the semiconductor analogy to suggest that nonrelativistic electrons, whether positive or negative energy, have a gravitational mass of m_g independent of their inertial mass; i.e., the gravitational mass (at least at nonrelativistic speeds), like spin and ES charge, is thus assumed here to be an intrinsic parameter. *The immediate consequence of this assumption is that positrons (holes) would then have a gravitational mass of* $-m_g$, *since they represent the absence of a negative-energy electron with gravitational mass* m_g.

In the degenerate Fermi sea, for every electron with momentum $\mathbf{p}$ and spin s, there is another electron with momentum $-\mathbf{p}$ and spin $-s$, so that the filled sea has no net momentum or spin. A vacancy in the state ($\mathbf{p}$, s) leaves the sea with a net momentum $-\mathbf{p}$ and spin $-s$, which may be ascribed to the hole. Since it is a vacancy in a state with negative energy $-E$, the hole is ascribed positive energy E. Thus, a negative-energy electron and its associated hole have opposite energies, momenta, and spins.

On the other hand, we have $E = m\gamma c^2$, while $\mathbf{p} = m\gamma\mathbf{v}$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, so that $\mathbf{p} = E\mathbf{v}/c^2$. If $-\mathbf{p}$ and $-E$ are the momentum and energy, respectively, of a negative-energy electron, then $+\mathbf{p}$ and $+E$ are the momentum and energy, respectively, of its associated hole; therefore, they have the *same velocity*. When acted upon by an external force, they also must then have the *same acceleration*. (This result is derived for Bloch electrons in a solid based on general wave-mechanical considerations.) From the above it follows that their gravitational masses would have to have opposite signs, since their inertial masses have opposite signs: $-GMm_g/r^2 = m^*a \Leftrightarrow -GM(-m_g)/r^2 = (-m^*)a$, where M is a test mass.

Since the inertial masses of a negative-energy electron and its associated hole have opposite signs, so must their gravitational masses. Neither antimatter nor negative-energy vacuum particles are considered by general relativity (GR), so that the situations described above are not within the original purview of the theory; but EP cannot have the generality assumed by Einstein if gravitational repulsion exists. It would be easiest to sweep repulsive gravity under the carpet by relegating it to the realm of negative-energy electrons (which also have no place in GR), thereby assuming that positrons and electrons are mutually attractive gravitationally as well as electrically; but it would be premature to reject repulsive gravity *a priori*, given that GR has yet to be successfully quantized, while the solid-state analog is manifestly quantum mechanical.

The approach to quantizing gravity usually begins with the Dirac equation, generalizing the metric tensor for curved space. In this paper, however, an approximation of the Dirac equation will be adapted to gravity by treating gravitational and inertial masses independently, as is the case in solid-state theory.

3. SEMICONDUCTOR MODEL

Based on quite general arguments first derived by Heisenberg (1931) [cited by Mattack (1976)], particle energy may be defined relative to any convenient value, such as the Fermi level. Suppose a particle has wave number $\mathbf{k}$, momentum $\hbar\mathbf{k}$, kinetic energy $\hbar^2k^2/2m$, and potential energy U, and that the arbitrary baseline energy W_0 exceeds $(\hbar^2k^2/2m) + U$; then the energy referred to this baseline, $E(\mathbf{k}) = (\hbar^2k^2/2m) + U - W_0$, is negative. Consider an intrinsic semiconductor (e.g., pure silicon or germanium): at absolute zero, no electrons populate energy states above the Fermi level (i.e., the "conduction band" is empty). The valence band, below the Fermi level, is completely full. The two bands are separated by an energy gap of ~1 eV. Conduction-band electrons are analogous to normal, positive-energy (positive-mass) electrons. Valence-band electrons in the Fermi sea are analogous to negative-energy (negative-mass) electrons populating the Dirac vacuum. The energy gap separating the two bands is equivalent to the forbidden region ($2mc^2$ wide) between the positive and negative rest energies. Pair production and annihilation may be achieved in both cases through the agency of photons.

Heating the lattice or irradiating it with sufficiently energetic infrared photons can lift electrons out of the valence band into the conduction band, leaving behind vacant Bloch states or "holes." A filled band does not contribute to the current, because an electron with wave number $\mathbf{k}$ is paired with another whose wave number is $-\mathbf{k}$. Thus, a vacancy (hole) in state $\mathbf{k}$ leaves an unpaired electron in state $-\mathbf{k}$, which contributes to the net current. The hole represents a positive contribution of $E_h(\mathbf{k}) = -E(\mathbf{k}) = W_0 - [(\hbar^2k^2/2m) + U]$ to the total energy, since the energy of the absent valence-band electron, $E(\mathbf{k})$, is negative. The energy contributions of conduction-band electrons, since they lie (by definition) above W_0, are positive.

For many phenomena, the potential energy (in momentum representation) may be usefully expanded as a function of k^2, absorbing the constant term into the baseline energy and neglecting terms of order higher than k^2. The k^2 term is absorbed into the kinetic energy, so that the energy appears to be that of a free quasiparticle with an effective mass m^*, where m^* is now defined for an isotropic energy band by $(1/m^*) = \hbar^{-2}(d^2E/dk^2)$. Positive effective mass is thus ascribed to conduction-band electrons, and negative effective mass to valence-band electrons near the Fermi level. The Dirac picture is similar: for nonrelativistic electrons (of positive or negative energy), the energy is quadratic in the momentum, besides the constant rest-mass term, so that negative "kinetic energy" corresponds to a negative inertial-mass increment, as it were. The contribution of the entire valence band may be characterized by its vacancies (if they are relatively few and near the top of the band), which have positive effective mass, positive effective charge ($+e$), and the opposite wave number (analogous to momentum) relative to the electrons that would have occupied those states.

If Bloch electrons are treated as wave packets, their group velocity is given by $\mathbf{v} = d\omega/d\mathbf{k} = \hbar^{-1}(dE/d\mathbf{k})$. On the other hand, for a "classical" particle treated relativistically, $E^2 = (pc)^2 + (mc^2)^2$; then $dE/d\mathbf{p} = \mathbf{p}c^2/E \approx \mathbf{p}/m \approx \mathbf{v}$ for $v << c$. Using the quantum mechanical identity $\mathbf{p} = \hbar\mathbf{k}$, the analogy is complete. In either case, a peculiarity of a negative-energy, negative-m^* Bloch electron is that its momentum and velocity point in opposite directions. Although velocity is defined differently for a Dirac electron, the same is true of negative-energy electrons. Intuitively, this is clear from $\mathbf{p}/m^* = \mathbf{v}$.

However, because a hole in state $\mathbf{k}$ has the wave number $-\mathbf{k}$ as well as effective mass $-m^*$ (here $m^* < 0$) and energy $-E$ (where $E < 0$), it follows that a negative-energy electron state and its corresponding hole have the same velocity, and, by extension, would have the same acceleration under an external force. Thus, under a Lorentz force due to external electric and magnetic fields, the equation of motion is $q(\mathbf{E} + (\mathbf{v}/c)\mathbf{x}\mathbf{B}) = m^*\mathbf{a}$. This is clearly invariant if both m^* and the charge q reverse signs together, i.e., a negative-energy electron and its corresponding positive-energy hole accelerate in the same sense, viz., like positrons—opposite the sense in which a positive-energy (conduction) electron responds.

Kittel (1971) says (pp. 326ff), "It has been established (see Chapter 11) by means of cyclotron resonance experiments with circularly polarized radiation on semiconductors that holes and electrons rotate in opposite senses in a magnetic field, as one would expect for charges of opposite sign. The radiation is absorbed by electrons for one sense of circular polarization and by holes for the opposite sense." Kittel also makes the point (pp. 331ff) that "[t]he crystal does not weigh any less if m^* is smaller than m, nor is Newton's second law violated for the crystal *taken as a whole* . . ." [Kittel's italics]; "m" is the electron mass, which is the same (except for negligible relativistic corrections) for electrons above ($m^* > 0$) and below ($m^* < 0$) the Fermi level.

What is of interest here is the influence of gravity on conduction-band electrons ($m^* > 0$), valence-band electrons ($m^* < 0$), and holes ($m^* > 0$). Newtonian gravity will suffice for this exercise. Now, the force on an electron due to gravity is $m_g\mathbf{g}$, where m_g is the *actual* (bare) mass of an electron, regardless of its effective mass within matter. In this sense the gravitational mass is analogous to charge, in that the Lorentz force, too, has the same form within condensed matter as it does in a vacuum; however, the relationship between force and acceleration (i.e., the effective inertial mass m^*) responds to the action of the ionic lattice on electrons, as the inertial mass (in a different way) varies with the velocity of the reference frame. It will be shown that in the case of a hole, $m_g < 0$.

Since electrons have the same gravitational mass whether their effective mass is positive or negative, it follows that for a given external field $\mathbf{g}$, $m_g\mathbf{g}$ has the same value for any electron. Thus, a field that *attracts* a conduction-band electron ($m^* > 0$, analogous to a normal, positive-energy Dirac electron) will necessarily *repel* a valence-band electron ($m^* < 0$, analogous to a negative-energy Dirac electron), in the sense that its acceleration will point away from the source of the field. The reason is the same as the explanation for the opposite response to the Lorentz force: Screening forces within the lattice, which are responsible for its negative m^*, have this net effect, reminiscent of the interactions with the lattice that result in a net attractive interaction between electrons in a Cooper pair in a superconductor. See, for example, Harrison (1979), Kittel (1971), Madelung (1978), or other texts on solid-state theory.

By the general arguments used to derive the (group) velocity, it follows that the velocity and acceleration of any hole corresponding to a valence-band state (for which $m^* < 0$) must also lead us to observe that it is *repelled* by a field that would attract a normal conduction-band electron. Because $m_g\mathbf{g} = m^*\mathbf{a}$, while $\mathbf{g}$ ($g = -GM/r^2$, M representing normal matter) and $\mathbf{a}$ point in the same direction, it follows that m_g and m^* have the same sign for conduction-band electrons; on the other hand, m_g and m^*, and thus $\mathbf{g}$ and $\mathbf{a}$, have opposite signs for valence-band electrons. For holes, by contrast, m^*, and of course $\mathbf{g}$, have the same sign as for conduction electrons, while $\mathbf{a}$ corresponds to the (backward) response of valence-band electrons; thus, m_g must be negative. This should not be surprising, given the assumption that all electrons see a force $m_g\mathbf{g}$; m_g for a vacancy should be opposite that of the occupied state, as with its other parameters ($\mathbf{k}$, m^*, E, spin). One is reminded that soda bubbles rise. . . .

Thus, this simplified Newtonian model suggests that massive, electrically neutral bodies of normal matter would gravitationally attract positive-energy electrons but gravitationally repel holes and positrons (because $m_g < 0$) as well as negative-energy electrons (because $m^* < 0$). Generalizing on this, neutral matter should polarize the vacuum gravitationally in the same sense that electrons do electrically (by repulsion of vacuum particles). Presumably, a neutral antimatter body would repel normal electrons but attract positrons, holes, and negative-energy electrons, so that antimatter should polarize the vacuum in the opposite sense. Because each normal electron is surrounded by a cloud of negative-energy electrons (as in QED), however, the repulsion of the cloud will be the only physically meaningful response (unless, perhaps, the electron is highly accelerated).

4. ANTIMATTER AND ANTIGRAVITY IN THE LITERATURE

It has yet to be conclusively demonstrated or observed whether antiparticles respond to gravity in the same sense as normal particles. Darling *et al.* (1992) discuss experimental tests of the weak EP, i.e., that all test particles with the same initial velocity fall with the same acceleration in a given gravitational field. Such investigations have been conducted at the elementary particle level during the last three decades, primarily by measuring charged particles in drift tubes, beginning with Fairbank *et al.* (1964). Although Fairbank and his co-workers at Stanford reported preliminary measurements of freely falling electrons and positrons, Darling *et al.* (1992, p. 241) state, "The positron version of the [drift tube] experiment has not been completed due to difficulties in a suitable source of slow positrons ... The antiproton drift-tube experiment, to be conducted at CERN, has not yet reached the operational stage."[6]

They further assert that the result reported by Fairbank's group in the 1960s that "appeared to demonstrate that free electrons could be satisfactorily shielded from most extraneous fields by enclosing them in an evacuated vertical copper drift tube cooled to 4.2K ... has been the subject of controversy, since theoretical expectations of the electric fields induced by the effects of gravity on the drift tube, and due to patch potential variations on its surface, appear to preclude such a measurement" (Darling *et al.*, 1992, p. 238).

There are currently plans (see, for example, Gabrielse *et al.*, 1988) to measure the gravitational response of neutral antihydrogen (a positron bound to an antiproton), which may employ an "atom trap" (Chu, 1990).[7] In such measurements, however, it is still difficult to slow antiproton and positron plasmas in ion traps enough to allow antiatoms to form (Darling *et al.*, 1992, p. 241, and references there).

Related theoretical models of predicted anomalous gravitational behavior of antimatter have appeared in the literature, especially since the discovery of the antiproton in the mid-1950s, though, in general, physicists today are unfamiliar with them. Some of these models are discussed in more detail in Appendix A. In particular, an intuitive model of "antigravity" behavior between matter and antimatter, whose consequences have much in common with those of the approach described in the present article, was proposed by Morrison and Gold (1957) [cited in Morrison (1958)], and more explicitly by Morrison (1958), motivated primarily by the obvious predominance of matter over antimatter in our region of spacetime.[8]

Morrison (1958, p. 368) says, however, "Since neither Lorentz covariance nor general relativity seem useful guides, these notions have not been put into any more mathematical form capable of demonstrating even the correspondence required with everyday physics in the limit where all matter

[6] For the difficulty of obtaining a source of slow positrons, see the subsequent discussions by Fairbank *et al.* and Henderson and Fairbank cited in Darling *et al.* (1992).
[7] The author first learned of Chu's work in discussions with P. Bender and D. Spergel.
[8] The author learned of Morrison's work after developing a very similar antigravity model independently.

is of one form [i.e., matter or antimatter]. Failing that, I cannot claim to have a theory, but only the physical framework into which such a theory must fit."

5. SOME CONSEQUENCES OF REPULSIVE GRAVITY

Before deriving a formal model of matter–antimatter gravitational interactions, it is worth noting that according to the scheme proposed here, gravitational and ES forces would work in opposite ways: Positive-energy particles with like ES charges repel, those with unlike charges attract; particles in mixed populations (e.g., plasmas) tend to surround themselves with oppositely charged particles, tending toward neutrality even on a microscopic scale, in a manner reminiscent of antiferromagnetic ordering. In contrast to this, positive-energy particles with like gravitational charges would be mutually attractive; those with unlike charges, mutually repulsive. In mixed populations, neutral matter and antimatter would tend to be surrounded by their own species (matter or antimatter), while aggregates of like species would tend to segregate themselves from aggregates of the opposite type.

These "tendencies" could in general become manifest only when gravity is the dominant interaction; it is possible that highly relativistic matter at Planck-scale densities, if it can exist, may behave similarly. It is not clear how particles in the vacuum would interact gravitationally in regions with little normal matter, but it is possible that they could be responsible for Hubble expansion at a rate higher than expected, based on attractive gravitation alone. A less esoteric consequence of the tendency of matter and antimatter to aggregate in spatially separated regions is that while magnetic fields can exist where ES fields vanish, this will not in general hold for their gravitational analogs: Gravitomagnetic fields will in general be produced by moving masses in regions where the much stronger static potential is unneutralized and unshielded, making detection of the latter far more challenging than the EM counterpart. See discussion of cosmological implications in Section 8.

6. GENERALIZED DIRAC EQUATION

The present paper is motivated by cosmological considerations, including the general absence of antimatter in the universe (as far as our indirect methods can detect) and what may be an anomalously high Hubble expansion rate (based on recent comparisons of the observed Hubble constant vs. the age of stars in certain regions of the universe[9] and by inadequately explored consequences of the well-known analogy between "holes" in Dirac and solid-state theories, leading to an apparent symmetry at the particle level in the Dirac equation. An appropriate formulation is suggested which illustrates that symmetry and satisfies Morrison's requirement that the theory correspond to results obtained for normal matter (at least at nonrelativistic speeds and relatively low densities and temperatures). Dirac's equation is generalized to include gravitational as well as EM potentials, comparable to a post-Newtonian approximation[10] for weak gravitational fields, low source velocities, and even lower test-particle velocities, so that terms of the second order or more in v/c may be neglected. In this formalism, gravitational field equations may be represented in a form resembling Maxwell's equations [see Forward (1961), updated in Braginsky *et al.* (1977)].

Gravitational repulsion of the type described here is a clear violation of EP, so that the methods of GR cannot be used self-consistently to describe gravitational interactions in general. On the other hand, for interactions involving only normal matter (or only antimatter), the results of GR are known to be empirically correct for a broad range of phenomena, and any more general theory of gravitation must correspond to GR in such cases. It is shown in Appendix B that when the particle velocity is sufficiently small, the gravitational force equation is analogous to the Lorentz force equation. It is argued there, by analogy to EM interactions, that because in the static case the Newtonian approximation of gravity resembles Coulomb's law, this method may be extended for a test particle (moving at nonrelativistic speeds, far from very massive sources) to include gravitational repulsion when the source and test particle are of different species, e.g., a positron in the field of the Earth, the Sun, or other nearby massive source composed of normal matter. Gravitational scalar (Φ_g) and vector ($\mathbf{A}_g$) potentials, like the equivalent EM 4-vector (Φ, $\mathbf{A}$), are inserted into the Dirac equation as part of a transformation of the energy-momentum 4-vector (the canonical 4-momentum) representing an electron in the presence of external EM and gravitational fields that are large compared with those produced by the particle itself.

[9] See Jacoby (1994) and references therein, in particular Pierce *et al.* (1994) and Freedman *et al.* (1994). Both used Cepheid variables (observed from Mauna Kea and via the Hubble Space Telescope, respectively) to perform "direct" measurements of distances to galaxies in the Virgo cluster; from redshift measurements they derived a Hubble constant H_0 of 87 $\pm$ 7 and 80 $\pm$ 17 km/sec/Mpc, respectively. The standard Big Bang model then estimates the present age of the universe as about 7 billion years. "In contrast," Jacoby says, "some stars are thought to be 16×10^9 years old. If H_0 is correct, then alleviating this age 'crisis' demands that at least one of the following be considered: stellar ages are too high, an accelerating force exists, or the standard, closed-Universe, Big Bang model is incorrect."

[10] See, e.g., Einstein (1921), where he presents an early version of the post-Newtonian approximation; Weinberg (1972) presents Einstein's later, more accurate approximation in Chapter 9 (see his ref. 1, p. 249).

For nearly stationary sources and test particles, the gravitomagnetic contribution may be neglected in comparison with the Newtonian component; it is included mainly to emphasize the symmetry between the EM and gravitational potentials in the approximation used here. The argument that follows, however, does not depend on this symmetry; indeed, it is sufficient to use Newtonian gravity to support the inferences regarding the expected gravitational response of antimatter; but it is useful to examine the consequences of this model in a domain where the equation is approximately covariant with gravity included as a correction to the canonical momentum.

Weak gravitational interactions can, in principle, be similarly treated in related equations, such as the nonrelativistic and relativistic Schrödinger (or "Klein–Gordon," KG) equations, or the London equation (Ross, 1983). Because the Dirac equation, unlike KG, is linear in both the energy and the momentum, these operators do not operate on the gravitational and EM field operators, which allows for considerable generality in their functional dependence; while KG, like the Dirac equation, has negative-energy solutions, these do not have the undisputed connection with antiparticles that they do in Dirac hole theory, and will not be treated in this paper.

The gravitational mass is decoupled here from the inertial mass, allowing the former to be handled in a way analogous to electric charge. Moreover, EP is not assumed as an axiom, unlike the usual approach to quantum gravity (see, for example, Kaku, 1993), and the familiar covariant equation, with its γ-matrix representation, is eschewed for the sake of clarity in favor of a form based on the one used by Dirac to deduce the existence of antimatter,

$$[(cp_0 - q\Phi - m_g\Phi_g) - \boldsymbol{\alpha} \cdot (c\mathbf{p} - q\mathbf{A} - m_g\mathbf{A}_g) - \alpha_0 mc^2]\psi = 0 \quad (1)$$

where $q = -|e|$ for an electron, $p_0 = i\hbar\partial/\partial(ct)$, $\mathbf{p} = -i\hbar$ **grad**, m_g is the gravitational mass (and $m_g\Phi_g$ the gravitational potential energy) of the particle, m is its inertial rest mass, ψ is a 4-component spinor wave function, and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and α_0 are 4×4 matrices satisfying Dirac's anticommutation relation, $\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}$ $(i, j = 0, 1, 2, 3)$, chosen in a representation, as Dirac (1958, pp. 273ff) suggests, so that α_1, α_2, α_3 are all real, while α_0 is imaginary. Dirac (1958, p. 257); (see also Appendix C of this paper) does this by exchanging the usual α_2 and α_0 (Dirac's α_m, often called β).

Note that "m" in the Dirac equation is neither the rest mass nor the inertial mass, but rather the coefficient of the operator which extracts the rest-mass component of the energy eigenvalue, viz., $+mc^2$ for positive-energy electrons or $-mc^2$ for negative-energy electrons. [Explicit references to negative rest mass in this sense may be found, for example, in Kramers (1964),[11]

[11] See, for example, Kramers (1964), p. 288: "The negative sign is, however, also possible and corresponds to a negative rest mass . . . We meet here the *famous appearance of negative rest masses in the Dirac theory* to which Dirac drew attention in his paper" [italics in the original].

Schiff (1968, Chapter 53, pp. 487ff), and Messiah (1961, Vol. II, Chapter XX, pp. 887ff), in addition to Dirac's use of the term; the term "negative kinetic energy" used by many authors is negative for the same reason, since it tends to $-mc^2$ as the particle velocity approaches zero. The meaning of such terms is unambiguous in Bloch theory, as described above.]

As a first approximation for $v \ll c$, we can assume $|m_g| = m$. Since the model in this paper does not depend on relativistic particle speeds, the present discussion will be limited to velocities for which the relativistic mass corrections are negligible. (Particles with vanishing rest mass moving as the speed of light, in particular neutrinos, may have to be handled differently; but in such cases, there is no "relativistic correction to the rest mass." See the discussion at the end of this section.)

Taking the complex conjugate of equation (1),

$$[(-cp_0 - q\Phi - m_g\Phi_g) - \boldsymbol{\alpha} \cdot (-c\mathbf{p} - q\mathbf{A} - m_g\mathbf{A}_g) + \alpha_0 mc^2]\psi^* = 0 \quad (1^*)$$

we can generalize Dirac's conclusion (Dirac, 1958, pp. 274ff) to include gravity: If ψ is a solution of the wave equation (1) corresponding to a negative value for the kinetic energy, $cp_0 - q\Phi - m_g\Phi_g$, then ψ^* will belong to a positive value for $cp_0 + q\Phi + m_g\Phi_g$: "It follows that each negative energy solution of (1) is the complex conjugate of a positive energy solution of the wave equation obtained from (1) by substitution of $-q$ for q [and $-m_g$ for m_g], which solution represents an electron of charge $+e$ [and gravitational mass $-m_g$] ..." moving through the given EM and gravitational fields. "... We assume that *nearly all the negative energy states are occupied*, with one electron in each state in accordance with the exclusion principle of Pauli. An unoccupied negative-energy state will now appear as something with a positive energy ... We assume that *these unoccupied negative-energy states are the positrons*" [Dirac's italics; expressions in brackets are the author's].

If the gravitational behavior of positrons is demonstrated experimentally to be consistent with EP (viz., if they are gravitationally indistinguishable from normal electrons), then the theory presented above is clearly wrong, while the theory that negative-energy electrons gravitationally repel normal electrons should be given serious consideration. If, on the other hand, electrons and positrons are shown to behave differently in free fall, then m_g may indeed be treated as a kind of charge, and this extension of Dirac theory may suggest a way to quantize the gravitational field by analogy with the EM field, at least in this approximation.

Regarding the connection between inertial mass (m) and m_g, if the Earth, the Sun, or another massive body made of normal matter is the source of the gravitational potentials (Φ_g, $\mathbf{A}_g$) in this example, then at nonrelativistic speeds, convention (including EP) normally requires assuming $m_g = m$. Although the two quantities are formally independent in (1), they are known empirically to be proportional (at rest) with a high degree of precision, $\sim 10^{-11}$, based on Dicke's improved version (1964–1967) of the Eötvös (1889) experiment,[12] the value of the coupling constant $(-G)$ having been chosen so that the inertial and gravitational masses would have the same magnitude. The sign of the gravitational mass charge for fermions and antifermions evidently corresponds to lepton or baryon number, which is conserved in pair annihilation or creation. [Incidentally, proton and antiproton inertial masses have been shown by Gabrielse *et al.* (1990) to be identical to 40 parts per billion, and their article refers to measurements of similar precision for electrons and positrons, and for K^0 and anti-K^0 mesons.]

The nature of positronium (an electron–positron "atom" analogous to hydrogen) indicates that the inertial mass of a positron does not differ sensibly from that of an electron, since the net radial force on the positron, $(m_{e+})v^2/r + (-e^2/r^2)$, would not vanish unless $m_{e+} = m_{e-}$. Other elementary particle interactions support the idea that inertia is insensitive to distinctions between particles and antiparticles, or matter and radiation. Without a gravitational interaction to lift the "weight degeneracy," a particle of gravitational mass m_g (where $|m_g| = m$), its antiparticle, and a photon with energy $h\nu = mc^2$ would all exhibit the same mass, viz., the inertial mass.

Even if neutrinos have zero rest mass (electron, muon, and tau-meson neutrinos differ from one another in some internal degree of freedom, analogous to "color" for quarks), they are expected to have gravitational mass $|m_g| = m = \text{energy}/c^2$, though the author knows of no observational evidence to support or refute this conjecture. Assuming $|\Phi_g/c^2| << 1$, so that the gravitational contribution to the total energy is negligible, then since neutrinos and antineutrinos are neutral fermions, the generalized Dirac argument seems to imply that they should have equal and opposite gravitational mass charges corresponding to their lepton number (or their helicity), according to

$$[(cp_0 - m_g\Phi_g) - \boldsymbol{\alpha} \cdot (c\mathbf{p} - m_g\mathbf{A}_g)]\psi = 0 \qquad (2)$$

though it has not been demonstrated that the gravitational mass may be substituted into the generalized Dirac equation in this way for a particle moving at $v = c$. Opposite gravitational mass signs, in addition to opposite helicities, would then be required to break the particle–antiparticle degeneracy. If neutrinos are found to have finite rest mass, that value could replace m_g above.

[12] R. H. Dicke, experiments reported between 1964–1967; see ref. 10, p. 21, in Weinberg (1972). R. von Eötvös' series of experiments began in 1889; see ref. 17, p. 21, in Weinberg (1972).

7. ANTIGRAVITY AND THE EQUIVALENCE PRINCIPLE

In the regions of space where either matter or antimatter predominates, such as our own, GR should provide an adequate mathematical description of gravitational interactions on a macroscopic scale, although its "simple" geometrical interpretation must be revised if gravitational repulsion exists. Matter–antimatter repulsion, a phenomenon not yet observed in nature, is an obvious violation of EP, since the behavior of matter or antimatter in a repulsive gravitational field would clearly differ from its behavior under nongravitational acceleration.

For instance, a small sample of antimatter in a freely falling (e.g., orbiting) laboratory made of matter (or vice versa) should necessarily appear to be accelerated (e.g., by gravity plus centrifugal force) when no otherwise measurable gravitational force is exerted on it, since the laboratory and the sample would respond to the same field in the opposite sense. Light should be deflected toward, and attracted to, a massive body of either species (if a photon is indeed its own antiparticle), whatever the composition of the laboratory; the symmetry of electron–positron annihilation, particularly in the singlet state in which two photons of equal energy are emitted in opposite directions (measured in the center-of-mass frame), polarized at 90° to one another, indicates no way to distinguish photon from "antiphoton."

It may be objected that the concept of gravitational repulsion between particles and antiparticles, while entertaining, is counterintuitive and unsupported by empirical evidence, though not to date refuted by experiment or observation. However, analogous to Euclid's Fifth Postulate, the EP, along with universally attractive gravitation, may be independent of the other laws of physics, particularly those demonstrated on a microscopic scale, including special relativity (SR) and quantum mechanics. Weinberg says (1972), "I believe that the geometrical approach has driven a wedge between general relativity and the theory of elementary particles." The EP deviates from Mach's principle, which Einstein originally intended to incorporate into GR, since, according to the EP, within a freely falling frame, physics should no longer be influenced by the average gravitational interaction with the universe.

Although the model presented here deviates from the EP, it is consistent with Mach's principle (generalized to include attractive and repulsive gravity). The geometrical model on which GR is based depends on universal gravitational attraction, which curves space in a way that assumes it to be an elastic continuum that responds to the local gravitational stress produced by the presence of matter-energy. Extending the quantum mechanical picture of the vacuum, which began as Dirac's sea of negative-energy electrons, we may reasonably expect massive bodies to polarize the vacuum gravitationally, much as charged particles do, though whether this interaction is predominantly attractive, repulsive, or even more complex depends on whether gravitational repulsion can be shown to exist, and if so, under what circumstances. In any case, it may provide a physical basis for the "curvature of space," possibly in the form of a variable index of refraction to which particles, antiparticles, and photons could respond differentially.

The independence of Euclid's Fifth Postulate gave birth to the Riemannian geometry that underlies Einstein's vision of spacetime; the next generalization may radically alter the way gravity is perceived, and how it may one day be quantized.

8. COSMOLOGICAL IMPLICATIONS

The intriguing possibility that matter and antimatter may repel one another gravitationally could provide a simple mechanism to explain aspects of such diverse cosmological phenomena as inflation, the general absence of antimatter in our region of the universe, and the development of certain large-scale structures, for example, while remaining consistent with the results of GR in our matter-dominated region of spacetime. A finite cosmological constant Λ in Einstein's equation is sometimes invoked as a potential source of global repulsion, but it does not enter GR in a way that unequivocally points to its source or dynamic nature. It is interesting to note in passing that a universe initially consisting only of fermion–antifermion pairs could of course produce photons, but the converse is not necessarily true: There is no experimental evidence that matter can be produced by photons without the presence of matter.

The Big Bang may have been driven by gravitational repulsion, perhaps between matter and antimatter (if there was an epoch during which densities corresponded to interparticle distances on the Planck scale) and/or between positive- and negative-energy matter. Assuming the relativistic increase in mass with increasing temperature and velocity, it is evident that matter–antimatter repulsion could overcome ES forces at "Planck densities," thus providing an engine for inflation in the very early universe. In the absence of radiation, nothing would effectively bind matter to antimatter. If radiation is present, inflation could also be driven by conventional radiation pressure, as opposed to, or in addition to, the negative radiation pressure assumed in conventional inflation theory (Peebles, 1993, p. 395). Matter and antimatter would naturally segregate. In any event, when gravitational repulsion predominates, gravitational deceleration must be reduced.

Once the universe becomes radiation-dominated, and at densities and temperatures where EM forces dominate gravity, large-scale aggregation of matter and antimatter (separately), and the segregation of those aggregates from one another, would probably be prevented until "[re]combination" and the decoupling of matter and radiation restored gravity's position as the dominant long-range force in the universe. Initial *N*-body simulations by the author confirm that such aggregation of matter with matter and antimatter with antimatter can occur, so that these aggregates move away from one another as the universe expands. This is the explanation suggested here for the almost total absence of antimatter in our part of the universe; no cosmic asymmetry between matter and antimatter is assumed.

As discussed above, the model proposed here predicts that despite the attractive form of the interaction between negative-energy electrons (and, independently, within the positively charged "lattice"), this interaction should result in pressure on particles within the vacuum to accelerate away from one another (assuming the vacuum as a whole is electrically neutral), in opposition to the tendency of positive-energy matter and antimatter (separately) to decelerate the expansion. Recent conflicting reports regarding the magnitude of the Hubble constant (see footnote 9) contrast the calculated age of older stars in the Galaxy ($\sim$16 billion years) with two independent measurements of the distance to Virgo and the associated red shifts, the latter yielding an age of the universe of only 7–8 billion years, based on Hubble's law. Clearly galaxies remain intact, so that gravity must suffice to hold them together against any vacuum expansion; and Newtonian gravity must include whatever effects result from long-range vacuum interactions.

APPENDIX A. RESPONSE TO ANTIMATTER AND ANTIGRAVITY IN THE LITERATURE

A1. Overview

Soon after Dirac's prediction of the existence of antimatter, the idea of negative mass was considered and rejected, mainly because negative inertial mass leads to apparent absurdities in particle dynamics. The possibility that gravitational repulsion could occur between matter and antimatter has surfaced from time to time in the physical literature, but is not considered explicitly in standard texts and contemporary monographs on gravitation and cosmology, such as Weinberg (1972), Misner *et al.* (1973), Peebles (1993), and Islam (1992). Morrison (1958) was an early, articulate proponent of the type of antigravity discussed in the present study. Although his work is well known to a cadre of quantum antigravity and fifth-force theorists [e.g., Nieto and Goldman; see their extensive review article (Nieto and Goldman, 1991)] and experimentalists (e.g., Darling *et al.*, 1992), it is not generally known in the physics community today, even among specialists in GR and gravitational theory, according to an informal poll by the author.

Certain theorists, particularly "steady-state cosmology" proponents,[13] have considered negative mass in a cosmological context. In Hoyle and Narlikar's (1974) model, particles of negative mass exist in a region of negative time, and are explicitly "not antiparticles, since particles and antiparticles always annihilate with nonzero energy release" (p. 178). In that model, gravitational and inertial masses have the same sign.

Darling *et al.* (1992, p. 238), explain:

> The original motivation for 'antigravity' faded in the late 1970's as several authors realized that, under certain conditions, the new grand unified theories could allow a baryon asymmetry to evolve during the early universe, thereby explaining the absence of antimatter ... Despite this, anomalous gravitational properties of antimatter were still being considered by some authors—Scherk (1979), Goldman and Nieto (1982), and Macrae and Riegert (1984)—who were working on quantum gravity theories based on supersymmetry ... Quite unlike the early notions of 'antigravity', such interactions are expected to produce a slightly larger downward acceleration for antiparticles in the Earth's gravity than for their counterparts. Goldman *et al.* (1987) estimate that antiprotons may fall a few percent faster than protons. [See references in Darling *et al.* (1992).]

These supersymmetric theories are far more complex than the heuristic model presented in the present paper, and will not be dealt with here. Since Morrison's scheme and the related objections to "classical antigravity" are intuitive and based on considerations directly relevant to some of those proposed here, they will be analyzed now in more detail.

A2. Morrison's Conjectures, Good's Objection

Morrison (1958) was primarily interested in baryonic matter as the principal contributor to gravitational interactions in the Universe. In his Richtmyer Memorial Address, he carried out *Gedanken*-experiments to illustrate certain unusual characteristics of his model of antigravity, subsequently discussed—and challenged—by a number of authors, e.g., including Schiff (1958, 1959), Good (1961), and Nieto and Goldman (1991).

Good's objection, based on the assumed behavior of neutral K mesons under gravity and antigravity, is discussed by Nieto and Goldman (1991, pp. 225, 258ff). Apart from their comments, it should be pointed out that the present article focuses on a generalization of the Dirac equation, intended for fermions, particularly leptons. K-Mesons, on the other hand, are not only "strange" bosons; both K^0 and anti-K^0, being mesons, are composed of strongly interacting quark–antiquark pairs. The meaning of "antiparticle" must thus be very different for mesons, and their gravitational behavior should

[13]For example, the review article by Bondi discusses inertial mass and active and passive gravitational mass [cited in Darling *et al.* (1992), Nieto and Goldman (1991), and Schiff (1958, 1959)]. F. Hoyle and T. Gold were also among the original steady-state cosmology proponents.

not be expected to be comparable to that of, say, electrons and positrons. Baryons, however, are spin-1/2 fermions composed of three spin-1/2 quarks (antiquarks for antibaryons), bound together by massless, spin-1 (i.e., bosonic) gluons, and may therefore be amenable to an extended form of the Dirac-based model proposed here.

Moving on to Morrison, he asserts in his first example (Morrison, 1958, pp. 367ff) that perpetual motion could be sustained by passing a photon back and forth between alternately excited states of matter and antimatter in an external gravitational field; to prevent this, photons must be attracted by matter and antimatter alike, and the "photonic" component of excited matter or antimatter should also be attracted gravitationally by both. However, while the behavior of real photons in the presence of a massive body is known, and is probably the same whether it is composed of matter or antimatter, it is not clear what is "photonic" about a photon once it has been absorbed by matter, since it has essentially been annihilated, just as an emitted photon is created.

The net effect of photon absorption or emission by a bound atomic electron is to change electronic binding and kinetic energies. The change in binding energy may be seen as a conversion of at least a portion of the photon's energy to (or from) energy stored in the atomic EM field, which may be represented by virtual photons. Morrison added (p. 368): "It seems most probable that the relativistic increase of inertial mass seen in relative motion corresponds to an increase in the sort of gravitational mass represented by photons. Then a[n] antinucleon moving very rapidly, near the speed of light, will have a gravitational behavior in the sun's field which is almost the same, and approaches in the limit, the behavior of a photon, a proton or any other object of equal inertial mass and velocity."

This poses a number of problems. First of all, SR makes no specific claim regarding the physical alteration of an object due to its motion relative to a particular observer. Suppose an observer accelerates (e.g., by firing a rocket motor) toward an originally stationary object, and then cuts the engine and coasts at a constant velocity. According to SR, the inertial mass of the object will then be seen by that observer to have increased. Has some "photonic increment" altered the gravitational or inertial mass of the object? Perhaps; but if so, it depends on some undocumented characteristic of the vacuum.

Moreover, relativistic fermions resemble photons primarily in that $E \approx pc$. It would be more edifying to compare the behavior of relativistic particles and antiparticles with that of (apparently) massless neutrinos and antineutrinos; such behavior, which has yet to be observed, may resemble the gravitational characteristics of massive fermions, as suggested by the Dirac analogy above. The path of a neutrino passing close to the Sun would be expected to coincide with that of a photon. An antineutrino may be deflected in the opposite sense; perhaps a distinction may be drawn between the component of deflection due to polarization of the vacuum and that due to the specific gravitational interaction between the massive source and the test particle or photon. The trajectory of the antineutrino should form the closest approach for relativistic antiparticles; if "neutrino" and "antineutrino" have been incorrectly designated, the descriptions of their trajectories should be exchanged.

Finally, if the annihilation or creation of photons results in a net change in the rest mass of matter in an ambient gravitational field, then the absorption or emission of real or virtual gravitons may be involved, compensating for the change in gravitational energy, as suggested in the discussion of pair annihilation below. In any event, the "photonic" nature of the relativistic mass increase is not self-evident, *pace* Morrison.

In a second example, Morrison (p. 368) presents a perpetual-motion paradox derived from the annihilation at a higher potential, and generation at a lower potential, of a particle–antiparticle pair. He writes, "Place in each pan of the dumbwaiter . . . a nucleon–antinucleon pair. Allow the pair in the upper pan to annihilate. The products can be reduced to photons. These photons plainly have a net gravitational mass, and the pan will fall. Work can be extracted. When the pan with photons has reached bottom, and [the] other is at the top, the photons could be used to reform a pair, and the system returned to its initial state, work having been extracted . . ."

It is not clear how free photons would contribute to the weight of a "pan"; we will let them interact with it, avoiding the complexity of gamma-ray-reflecting mirrors by letting the photons be absorbed without recoil by nuclei embedded in a cooled, massive crystal lattice, like ideal Mössbauer-effect absorbers. If the lattice is now heavier by the effective mass of the photons, the pan will fall in the external gravitational field, and can be made to do work before coming to rest. If the energy levels of the absorbing nuclei remain constant during this process, then the gamma rays may be reemitted by a similar idealized process with the same energy as before.

Now suppose that the pair in the upper pan also annihilates and the photons produced are absorbed as described above. If the pair of photons in the lower pan can produce an electron–positron pair as Morrison suggested, and if the "weight" of this pair should indeed vanish, then the lower pan will rise, and a *perpetuum mobile* will have been produced. However, it must be pointed out that there is no direct experimental evidence that a particle–antiparticle pair can be produced by a *pair* of photons in the manner described by Morrison; pair production involves a *single* gamma ray (of energy $> 2mc^2$) and a nucleus, interaction with which serves to conserve momentum. It is not self-evident, therefore, that Morrison's scenario is physically realizable.

Morrison assumes that all energy absorbed or emitted in his model will be photonic in nature, so that the photons emitted at the "higher" potential will be less energetic than those required for pair formation below, in order to prevent perpetual motion. This is not manifestly impossible, and may be tested when the energy discrimination of gamma-ray detectors is sufficiently precise. The ratio of this gravitational transition to the annihilation energy is $GM_E/rc^2 = gr/c^2 \approx 7 \times 10^{-10}$, or $\sim 7 \times 10^{-4}$ eV (or ~ 0.7 eV for proton–antiproton "nucleonium" annihilation), if r is the radius of the Earth, which may be measurable effects, like the 8.5×10^{-4} eV transition of spin–spin coupling in positronium from the ^1S ("para") state to the ^3S ("ortho") state. [See Pirenne (1946, 1947) (cited in Heitler, 1954, p. 274) and Berestetzky and Landau (1949) (cited in Heitler, 1954, p. 274) regarding positronium.] Perhaps the angular correlation of annihilation radiation (ACAR), i.e., the pair of gamma-rays emitted following positronium annihilation, might be refined sufficiently to measure the gamma-ray energies and the angle between them at very different gravitational potentials. [See Debenedetti *et al.* (1950) regarding ACAR.[14]]

Alternative theoretical approaches to this problem that do not predict different gamma-ray energies at different gravitational potentials usually postulate the existence of gravitons to account for the presumed requirement for greater pair-formation energy at the "lower" potential. If a single graviton is emitted, then the ACAR results should show a slight deviation of the photon trajectories from 180° in their center-of-mass frame; but if a pair of gravitons is emitted with equal and opposite momenta, the photon angle may not reveal their presence; graviton detectors would then be needed to resolve the issue. Nieto and Goldman (1991, pp. 250ff) discuss Morrison's "machine" from the standpoint of coupling via tensor, vector, and scalar gravitational fields. In the present paper, a simplified paradigm is suggested.

Consider the following scenario: Suppose that when a particle–antiparticle pair annihilate, not only are photons produced whose total energy equals the equivalent inertial mass of the matter–antimatter pair (minus the ES binding energy of the positronium); but also, one or more gravitons (of some unspecified type) are emitted whose total energy equals the difference between the combined gravitational potential energy of the particle–antiparticle pair (=0 in this simple model) and that of the photons produced by their annihilation (this potential energy should be negative, assuming that photons are attracted by matter and antimatter alike; in this case, $-2GM_Em/r_1$,

[14] Discussed in McGervey (1971), pp. 446ff. Positronium annihilation is primarily used to map the Fermi surfaces of solids, including high-T_c superconductors; e.g., see Wachs *et al.* (1989).

where r_1 is the distance from the center of mass of the Earth to the altitude at which annihilation occurs). In this instance, as with the Fermi sea, it is the change in the potential energy, i.e., in the integral of the force on the test mass from infinity to the point of measurement, that is significant here, not its absolute value (S. Kutter, private communication; see also Nieto and Goldman, 1991, p. 258). Since we are currently discussing a frame of reference within the solar system, force can be determined directly from acceleration without ambiguity.

In order to produce a particle–antiparticle pair now at this lower altitude, a sufficiently energetic photon (instead of the photon pair produced earlier, for the reasons stated above) can be converted into a particle–antiparticle pair (1) only if it interacts with a massive body (e.g., a nucleus) and (2) only if one or more virtual gravitons can be absorbed from the ambient field to create the pair at this new potential, which, while lower for the particle and the photons, is now higher than before for the antiparticle; thus, more gravitational energy (which we may call the "gravity-flip energy," $2GM_Em/r_2$, where r_2 is the distance from the center of mass of the Earth to the lower altitude at which pair production occurs) must be absorbed from the "reservoir" (i.e., the ambient field) at the "lower" potential than is liberated at the "upper" potential, so that entropy is increased, and no violation of either the first or the second law of thermodynamics results. This picture implies that the magnitude of the local gravitational potential and potential energy differences between different gravitational states is in principle an empirical question. Should it be shown that distinct gravitational charges do exist, then reactions of this type could in principle be used to construct a graviton detector—or a gravitational refrigerator.

A3. Schiff's Challenge and Virtual Pairs

Soon after Morrison and Gold's antigravity hypothesis was published, Schiff (1958, 1959) deduced from the results of the Eötvös experiments that positrons and electrons have the same gravitational mass, based on putative contributions to the gravitational mass of the samples by virtual electron–positron (and nucleon–antinucleon) pairs produced by their nuclear Coulomb fields. Authors from Morrison, anticipating Schiff (Morrison, 1958, p. 367), to Goldman and Nieto (1982) have asserted that Schiff's indirect argument, based on virtual particles, hardly has the weight of experimental evidence appropriate to real electrons and positrons, and Darling *et al.* 1992, pp. 237ff point out that Morrison and Gold would have included the virtual pairs in the EM, "photonic" part of the mass, which would be attractive to matter and antimatter alike, leading to null Eötvös results.

The latter objection to Schiff's argument may be expanded as follows: In quantum electrodynamics (QED), there is a finite probability that a virtual photon associated with the atomic Coulomb field will decompose into a virtual electron–positron pair, a fluctuation of the Fermi sea carrying the same net momentum as its parent virtual photon; similarly, virtual pairs annihilate to form virtual photons. It should be noted here that vacuum polarization is a corollary of Dirac's picture of the electron "vacuum" as an infinite sea of states filled with negative-energy electrons. Vacuum polarization is manifested by the average distribution of an electron's charge in space, an observable quantity. In QED, "virtual" particles and photons have finite lifetimes shorter than the time required to measure their energy, according to Heisenberg's uncertainty principle, $\Delta t \lesssim \hbar/\Delta E$.

A free electron–positron pair can be created only by absorption of energy (from a real gamma ray or the kinetic energy of a charged-particle impact) sufficient to separate them, viz., $E > 2mc^2$. Components of virtual electron–positron pairs, unlike real electrons and positrons, do not exist independently of each other—they are created and separated in the vacuum only to annihilate into other virtual photons, and are (in a sense different from quarks) confined to existence as transient dipoles, the "alter ego" of virtual photons: The wave functions of the virtual electron and positron must always overlap enough to allow for their mutual annihilation. Since $c\Delta t \lesssim c\hbar/\Delta E$, and $\Delta E > 2mc^2$ for the virtual photon and/or the pair together, while the photon energy $\Delta E = 2\pi\hbar c/\lambda$, then the separation of the pair cannot exceed the photon and/or Compton wavelength. One may plausibly argue that any photon of sufficiently high energy may be decomposed into virtual particle–antiparticle pairs in this way, since the pairs always remain "within" the photon, as defined by its wavelength. Insofar as the macroscopic gravitational behavior of photons is known, then, a virtual pair may be expected to act as a single [virtual] photon.

The conclusion is that Schiff correctly inferred that the inertial and passive gravitational masses of virtual pairs are the same to within the experimental error of the Eötvös experiments; but he did not establish beyond a reasonable doubt that the inferred behavior of virtual electron–positron pairs in a gravitational field is a reliable measure of the gravitational behavior of real, free electrons and positrons.

APPENDIX B. APPROXIMATE GRAVITATIONAL POTENTIALS USED IN THIS PAPER

Braginsky *et al.* (1977), following Forward (1961), use a scalar gravitational potential Φ_g and a vector potential $\mathbf{A}_g$ to generate "electric" and "magnetic" type gravitational fields $\mathbf{g}$ and $\mathbf{H}_g$, respectively, where $\mathbf{g} = -\mathbf{grad}\,\Phi_g - (1/c)\partial\mathbf{A}_g/\partial t$ and $\mathbf{H}_g = \mathbf{curl}\,\mathbf{A}_g$. Using a Lorentz-like gauge, $\mathbf{div}\,\mathbf{A}_g = -(3/c)\,(\partial\Phi_g/\partial t)$, we obtain the associated Maxwell-like field equations for GR

$$\text{div}\,\mathbf{g} = -4\pi G\rho_0[1 + 2(v^2/c^2) + (\Pi/c^2) + 3p/\rho_0c^2] + (3/c^2)(\partial^2\Phi_g/\partial t^2)$$
$$\text{curl}\,\mathbf{g} = -(1/c)\partial\mathbf{H}_g/\partial t$$
$$\text{div}\,\mathbf{H}_g = 0$$
$$\text{curl}\,\mathbf{H}_g = -16\pi G\rho_0\mathbf{v}/c + (4/c)\,\partial\mathbf{g}/\partial t$$

where ρ_0 is the baryon density in the local rest frame, $\mathbf{v}$ is source velocity, Π is the specific internal energy (i.e., energy per baryon), and p is the radiation pressure. For a particle of velocity $|\mathbf{v}_0| < 10^5$ cm/sec, Braginsky *et al.* approximate the gravitational force on a unit mass [for GR, with typographical error in Braginsky *et al.* (1977), equation (3.10), p. 2054, corrected] as $\mathbf{F}/m_g = [1 + \frac{3}{2}(v^2/c^2)]\mathbf{g} + (\mathbf{v}_0/c)\times\mathbf{H}_g$, which reduces to $\mathbf{F} = m_g\mathbf{g} + \mathbf{v}_0\times[\mathbf{curl}(m_g/c)\mathbf{A}_g]$ if the particle velocity is so small that the second-order term can be neglected, viz., if $|\mathbf{v}_0| << |\mathbf{v}| << c$ (the reason for this condition will be explained below). This equation is now analogous to the Lorentz force; thus, the gravitational field components should transform the 4-momentum as the EM 4-potential does, so that $p_\mu \rightarrow p_\mu - (q/c)A_\mu - (m_g/c)A_{g\mu}$.

The formulation described above is based on EP, where $m = m_g$. No distinction is made between matter and antimatter, nor between particles and photons. It is reasonable to assume that it is correct for matter–matter and antimatter–antimatter interactions, and for photons interacting with a massive matter or antimatter source. It is argued here that for nonrelativistic masses, the EM analogy may be inverted to show that if the gravitational mass (m_g) is treated as a charge, nominally independent of the inertial mass (m), then a repulsive Lorentz-like gravitational interaction can be derived for fermion–antifermion interactions. This is based on the fact that, in the Newtonian limit (at low velocities, sufficiently far from a noncompact source), the law of gravity closely approximates Coulomb's law. A small Lorentz "boost" to a static field, $\mathbf{g}_0$ perpendicular to the boost velocity $\mathbf{v}$ produces a gravitomagnetic field perpendicular to $\mathbf{v}$, approximately equal to $\mathbf{H}_g = \gamma[(\mathbf{v}/\mathbf{c}) \times\mathbf{g}_0]$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, as well as an "electric" component, now $\gamma\mathbf{g}_0$, independent of the "test charge" (m_g).

In this approximation, the field producing the force on a "test charge" moving with velocity $\mathbf{v}_0$ is just the Lorentz boost of the field acting on a static "charge" (i.e., Newtonian gravity): $\mathbf{F}/mg = \mathbf{g} + (\mathbf{v}_0/c)\times\mathbf{H}_g$. If m_g transforms as the inertial mass does (unlike electric charge, which is a Lorentz invariant), then the field equation should be multiplied by $(1 - v_0^2/c^2)^{-1/2}$. If the same massive body is the source of both the gravitoelectric field $\mathbf{g}$ and the gravitomagnetic field $\mathbf{H}_g$, and if the source velocity $\mathbf{v}$ satisfies the condition $|\mathbf{v}_0| << |\mathbf{v}| << c$, as required above, then it is possible to neglect terms of order $(v_0/c)^2$ or smaller, leading to the same approximate force law as the one derived by Braginsky *et al.*

APPENDIX C. REPRESENTATIONS OF DIRAC MATRICES USED IN THIS PAPER

The α_0, α_1, α_2, and α_3, are 4×4 matrices chosen as follows:

$$\alpha_0 = \begin{vmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{vmatrix}, \qquad \alpha_1 = \begin{vmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{vmatrix}$$

$$\alpha_2 = \begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix}, \qquad \alpha_3 = \begin{vmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{vmatrix}$$

Their order is somewhat arbitrary in the example here, except that the four must anticommute, $\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}$ $(i,j = 0, 1, 2, 3)$, and α_0 is purely imaginary, while the other three are real, based on the 2×2 Pauli spin matrices and the identity matrix shown below:

$$\sigma_1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \qquad \sigma_2 = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}$$

$$\sigma_3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}, \qquad I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

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